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LOW x_B PHYSICS with OPEN EYES

E. Levin

Fermi National Accelerator Laboratory, P.O.Box 500, Batavia, Il 60510, USA
and
St. Petersburg Nuclear Physics Institute, 188350, Gatchina, RUSSIA

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ABSTRACT

This is a status report on low x_B physics in deeply inelastic processes just after the first experimental data from HERA. The talk has two goals: i) to discuss what we have learned about physics from HERA data and ii) to give a brief review of the recent theoretical development in the region of small x_B in deeply inelastic scattering.

1 Introduction

I am viewing this talk as a summary of what we have learned about physics in the region of low x_B just after the first experimental data from HERA. I hope it will be an honest review of our hopes and attempts to understand what is going on in the region of low x_B, which we have started for the first time to study experimentally.

The outline of my talk looks as follows:

1. Strategy and hopes.

2. Cold shower of the experimental data.

3. On the way to an analytic solution.

The title of this subsection I stole from David Gross who in his Cornell Summary gave ten predictions for the year 2008, the second one of which was:

> *" Analytic treatments in QCD will be developed to describe small x_B physics, Regge behaviour and hadronic fragmentation functions".*

4. Tragedy of low x_B.

Thus the second title of my talk could be *"from strategy to tragedy in low x_B physics"*.

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2 Strategy and hopes.

2.1 The map of QCD.

Fig. 1 shows the map of QCD as we understand it now. We can see three very distinguished regions of QCD with quite different physics and different level of understanding. However before we discuss these three regions in more detail let us introduce notations and necessary definitions.

In Fig.1 r is the distance that can be resolved by our microscope and due to uncertainty principle this distance is of the order of $\frac{1}{Q}$ where Q is the transverse momentum of recoiled electron in DIS. The second kinematical variable that we can introduce for such constituents is the fraction of energy (x_B) that carries it with respect to the hadron (proton for our example). If N is the number of gluons [1], the gluon structure function tells us what the number of gluons is with a definite value of $y = \ln \frac{1}{x_B}$, i.e.

$$x_B G(x_B, Q^2) = \frac{dN}{d \ln \frac{1}{x_B}} .$$

However it is more convenient to introduce the gluon density (ρ) in the transverse plane which we put on the vertical axis in fig.(??):

$$\rho = \frac{x_B G(x_B, Q^2)}{\pi R^2} \qquad (1)$$

where R is the radius of hadron.

In fig.(??) you can see three different regions:

1. *The region of small ρ at small distances r (low density (pQCD) region).*

This is the region where we can apply the powerful methods of perturbative QCD since the value of the running coupling constant $\alpha_s(r^2)$ is small here ($\alpha_s(r^2) \ll 1$).

2. *The region of large distances (npQCD region).*

Here we have to deal with the confinement problems of QCD, since $\alpha_s(r^2) \gg 1$. Thus in this kinematical region we need to use the methods of nonperturbative QCD.

3 *The region of small distances but high density of partons (hdQCD region) .*

This is our region of interest since here we have a unique situation where the coupling constant $\alpha_s(r^2)$ is still small but the density is so large that we cannot use in this kinematical region the usual methods of perturbation theory. So we have to conceive of something new to study this kinematical region. Already now I would like to stress that furtunately we can treat this region by approaching it from the low density QCD region. The description of methods developed to study this region on the border and new physical phenomena that we anticipate here is the main subject of this talk.

[1] I'll show a little bit later that the number of gluons increases in the region of small x_B and that all physics in this kinematical region is closely related to this fact. It is the reason why I am concentrating on discussion of the gluon density here

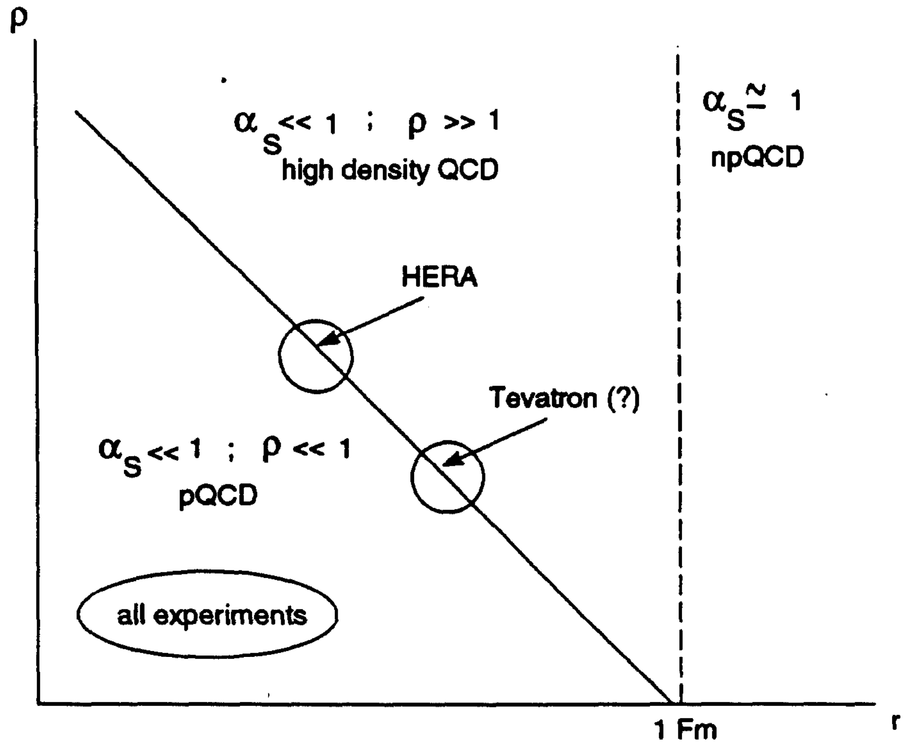

Figure 1: The map of QCD. ρ is the density of partons (gluons) in transverse plane (see eq.(1)) and r is the distances resolved in an experiment.

2.2 The low density (pQCD) region.

This is a region with small distances (very large value of transferred momentum Q^2 in hard processes such as deep inelastic scattering) and moderately small values x. The physical processes reveal here properties typical for hard processes. Namely:

1. The cross section (for example for virtual photon absorption in deep inelastic scattering) is very small, $\sigma(\gamma^* N) \ll \alpha_{e.m.} \cdot \pi R_h^2$ where πR_h^2, is a typical area of the hadron. It decreases as inverse power of Q^2 at large values of Q^2 ($\sigma(\gamma^* N) \propto \frac{1}{Q^2}$).

2. We have the transparent physical language to discuss physics here , namely, the parton language, especially conceived for hard processes.

3.In this region we can apply the leading log approximation (LLA) of perturbative QCD, which leads to a linear evolution equation for deep inelastic structure function (so called the Gribov-Lipatov- Altarelli-Parisi evolution equation [1]) and all properties of the GLAP equation are known quite well.

All experiments exept at the Tevatron and HERA checked precisely this kinematical region. As a result I dare say that during the last twenty years the **basics of QCD** have been studied and confirmed experimentally .

2.3 Nonperturbative QCD region.

In this kinematical region we also know a lot about QCD, mostly because a new kind of experiment has been contrived, namely so called lattice computer calculations. This is perhaps the cheapest way to study the main properties of confinement of quarks and gluons starting directly from QCD Lagrangian. The success of this approach is really remarkable and now lattice QCD is able to describe the spectrum of observed hadron whithin accuracy compatible with the experimental data (see review of A.S. Kronfeld and P.B.Mackenzie [2] for detail discussion of all relevant problems). However the main shortcoming of lattice QCD is the fact that we cannot apply it to scattering processes at the moment. Unfortunately, the same is true for another method which is not as general as lattice calculation but works for great variety of processes as a first approximation. We mean the sum rules of QCD [3]. The QCD sum rules are able to describe the property of confinement in average but this method uses the additional assumption that vacuum expectations of all operators are smaller in appropriate units than the typical scale in hadrons. In spite of all these difficulties the situation in this kinematical region is not so bad as in the high density region of QCD (see fig. (??)).

2.4 High density QCD.

In this kinematical region we are dealing with a system of partons which are still at small distances where the coupling constant of QCD α_s is still small but the density of partons becomes so large that we cannot apply here the usual methods of perturbative QCD. In fact the kernel of all theoretical problems here is also nonperturbative but the origin of nonperturbative effects here is quite different from that in the region of npQCD which we have discussed. Here we face the situation where we have to develop some new methods in order to deal with the dense relativistic system of gluons in a nonequalibrium state. Clearly we need to find new methods of quantum statistics that will allow us to describe theoretically such a system of partons. Unfortunately we are only at the beginning of this road.

The good news is the fact that we can approach this region theoretically from the pQCD region and in some transition region on the border of pQCD and hdQCD regions we can study this remarkable system of partons in many details. To illustrate what new physics we can expect in this transition region let us compare the behaviour of deeply inelastic scattering in this region with the pQCD region.

In the transition region the situation changes crucially:

1. The total cross section $\sigma(\gamma^* N)$ becomes large and, near the border with the hdQCD domain, even compatible with the geometrical size of the hadron at small x . It means that $\sigma(\gamma^* N) \rightarrow \alpha_{e.m.} \cdot \pi R_h^2$. In this kinematical region it smoothly depends only on $\log Q^2$, i.e. $\sigma(\gamma^* N) \propto F(\log Q^2)$.

2. The parton language can be used to discuss the main properties of our process but the interactions between parton become important. This interaction induces substantial screening (shadowing) corrections.

3. Fortunately, in this particular kinematical region the screening corrections are under theoretical control. We can go beyond the usual linear evolution equation and write the correct evolution equation, which becomes nonlinear. We will discuss this nonlinear equation a bit later.

2.5 How to penetrate the high density QCD region.

Now we would like to emphasize that this interesting kinematical region is easily reached in our scattering processes. In fact we know two ways to obtain a system of partons with a large value of density.

1. The first is godgiven, since we have sufficiently large and heavy nucleii. In ion - ion collisions we can reach a sufficiently high density of partons already at not so very high energies because the partons from different nucleons in a nucleus get freed .

2. Hard processes in hadron - hadron collisions or in deep inelastic scattering also give us an access to high density of partons because we expect a huge increase in the parton density or in other words the deep inelastic gluon structure function in the region of small Bjorken x. We will discuss all these expectations in the next section. However already at this point we would like to point out that the new experimental data from HERA [4] show that we have about 30 -50 gluons in a proton at $x = 10^{-4}$. This is a big number which we can compare with the number of nucleons in a nucleus of iron.

3. Of course we can use the hard processes in ion - ion collisions to utilize the both effects: increase of gluon density and a large number of nucleons in a target.

2.6 Nonlinear (GLR) evolution equation.

As we have discussed before the main new processes that we have to take into account in the region of hdQCD are parton - parton interactions. To incorporate such processes in our consideration we have to think of some new small parameter that controls the accuracy of our calculations. It turns out that such new small parameter [6] is equal to

$$W \;=\; \frac{\alpha_s}{Q^2} \cdot \rho \;. \tag{2}$$

The first factor in eq.(2) is the cross section for gluon absorption by a parton from the hadron. So it is clear that W has a very simple physical meaning, namely it is the probability of parton (gluon) recombination in the parton cascade. We can rewrite the unitarity constraint (??) in the form

$$W \;\leq\; 1 \;. \tag{3}$$

Thus W is the natural small parameter in our problem. It is worthwhile to note that W can be rewritten through the so called packing factor

$$PF \;=\; \langle\, r^2_{constituent} \,\rangle \cdot \rho \;. \tag{4}$$

Indeed

$$W = \alpha_s \cdot PF . \tag{5}$$

Using this small parameter we can resum the whole perturbative series (see below about phylosophy and strategy of resummation). The result of the resummation which has been done in ref. [6] can be easily understood considering the structure of the QCD cascade in a fast hadron. Inside the cascade there are two processes that are responsible for the resulting number of partons:

$$Emission\ \ (1 \rightarrow 2);\ \ Probability \propto \alpha_s \rho\ ; \tag{6}$$

$$Annihilation\ \ (2 \rightarrow 1);\ \ Probability \propto \alpha_s^2 r^2 \rho^2 \propto \alpha_s^2 \frac{1}{Q^2} \rho^2 ,$$

where r^2 is the size of produced parton in the annihilation process. For deep inelastic scatterinq $r^2 \propto \frac{1}{Q^2}$.

It is obvious that at $x_B \sim 1$ only the production of new partons (emission) is essential since $\rho \ll 1$, but at $x_B \rightarrow 0$ the value of ρ becomes so large that the annihilation of partons that diminishes the total number of gluons enters into the game.

Finally this simple parton picture allows to write an equation for the density of partons that takes these processes properly into account. Indeed, the number of parton in a cell of the phase space ($\Delta y = \Delta \ln \frac{1}{x_B}, \Delta \ln Q^2$) increases due to emission and decreases as result of annihilation. As an outcome the particle balance for this cell looks as follows:

$$\frac{\partial^2 \rho}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} \rho - \frac{\alpha_s^2 \gamma}{Q^2} \rho^2 , \tag{7}$$

or in terms of the gluon structure function $x_B G(x_B, Q^2)$

$$\frac{\partial^2 x_B G(x_B, Q^2)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} x_B G(x_B, Q^2) - \frac{\alpha_s^2 \gamma}{Q^2 R^2} (x_B G(x_B, Q^2))^2 . \tag{8}$$

Eq. (8) is the socalled GLR equation [6]. Unfortunately even now we need some complicated technique of summation of Feynman diagramms in W^n'th order of perturbation theory to calculate the value of γ [7] and to understand the kinematical region where we can trust the equation (8). The value of γ calculated in ref.[7] reads

$$\gamma = \frac{81}{16}\ \ for\ N_c = 3.$$

2.7 Our expectations.

Now we can summarize what new phenomena we anticipated in the region of small x_B just before HERA started to operate.

1. Increase of the parton density.

Both evolution equations (GLAP and BFKL) predict an increase of the parton density:

$$\mathbf{GLAP:} \quad x_B G(x_B, Q^2) \;\rightarrow\; e^{\sqrt{\frac{4\alpha_s}{\pi}\ln\frac{1}{x_B}\ln\frac{Q^2}{Q_0^2}}} \;;$$

$$\mathbf{BFKL:} x_B G(x_B, Q^2) \;\rightarrow\; x^{-\omega_0} e^{-\frac{\ln^2\frac{Q^2}{Q_0^2}}{\Delta \ln\frac{1}{x_B}}} \; where \; \omega_0 \;=\; \frac{4N_c}{\pi}\ln 2\alpha_s(Q_0^2) \,.$$

2. Growth of the typical transverse momentum of partons.

This property changes crucially the physics of deeply inelastic scattering at low x_B since the typical "hard" process occurs only at Q^2 larger than the mean value of the transverse momentum of partons. If Q^2 is smaller we face the very unusual situation where processes with the typical properties of "soft" interactions occur at small distances and can be treated by pQCD.

The growth of the typical parton transverse momentum is a common feature of the BFKL eqution as well as the GLR one. Indeed:

$$\mathbf{BFKL:} \ln\frac{\langle|p_t^2|\rangle}{Q_0^2} \;=\; \sqrt{\alpha_s(Q_0^2) 14\zeta(3)\ln\frac{1}{x_B}} \;;$$

$$\mathbf{GLR:} \quad \ln\frac{\langle|p_t^2|\rangle}{Q_0^2} \;=\; \sqrt{a\ln\frac{1}{x_B}} \,.$$

The constant a has been calculated in ref. [6].

3. Saturation of the gluon density.

Directly from the GLR equation we can see that the parton (gluon) density reaches a limiting value at low x_B. Inspite of the fact that we cannot trust the GLR equation at very small values of x_B we believe that the saturation of the gluon density is a new phenomena which reflects the basic property of the parton cascade in hdQCD.

Fig.2 shows all our expectations in terms of the gluon structure function.

3 Cold shower of the experimental data.

3.1 25 nb^{-1} experimental data.

Let us first summarize the first experimental result at small x_B from both collaborations at HERA (refs. [4]):

1. The increase of the value of the deep inelastic structure function F_2 at $x_B \rightarrow 10^{-4}$ shows that the gluon density reaches a sufficiently large value, namely

$$x_B G(x_B, Q^2) \;\rightarrow\; 40 - 50 \; at \; x_B \;=\; 10^{-4} \; and \; Q^2 \;=\; 20\, GeV^2 \,.$$

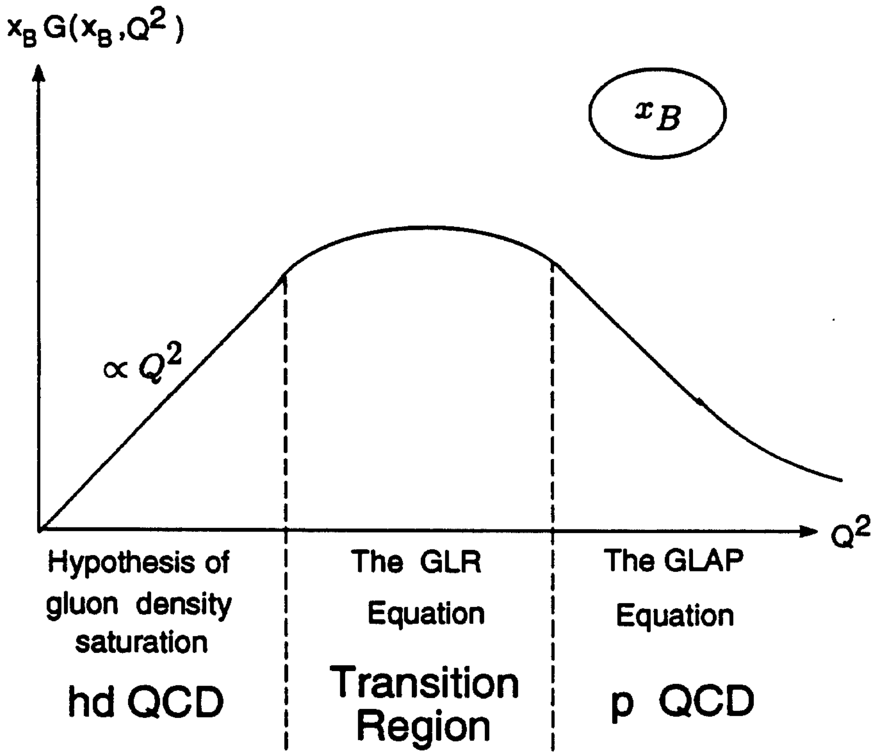

Figure 2: The behaviour of $x_B G(x_B, Q^2)$ versus Q^2 at fixed x_B.

2. The ZEUS collaboration measured a very important value, namely the diffraction dissociation cross section and the result is

$$\frac{\sigma^{DD}}{\sigma_t} = \frac{5.2nb}{80nb} = 6\,10^{-2}\, at\, x_B = 10^{-4}\; and\; Q^2 > 10\, GeV^2 \,.$$

3. F_2 experimental data can be described by the GLAP evolution equation assuming that at the initial virtuality $Q_0^2 \sim 5GeV^2$ the gluon structure function rapidly·increases at $x_B \rightarrow 0$ as $x_B G(x_B, Q_0^2) \propto x_B^{-\omega_0}$ and $\omega_0 \geq 0.3 - 0.5$.

4. Both collaborations measured the total cross section for photoproduction in the region of very small virtualities of the incoming photon and for a real photon. The result is that this cross section reveals the main property of "soft" processes, namely $\sigma_t \rightarrow s^{-\Delta}$ where $\Delta \sim 0.08$.

3.2 The value of SC from HERA data.

Let me write the deep inelastic structure function in the form:

$$F_2(x_B,Q^2) \;=\; F_2^{GLAP}(x_B,Q^2) \;-\; \Delta F_2(x_B,Q^2)\;, \tag{9}$$

where F_2^{GLAP} is the solution of the usual (GLAP) evolution equation and ΔF_2 is the SC. The ZEUS data on diffraction dissociation give us the possibility to estimate the value of SC . Indeed, it was shown in ref. [8] that we have a relationship betweeen SC and DD cross section directly from AGK cutting rules [9]:

$$\left|\frac{\Delta F_2(x_B,Q^2)}{F_2(x_B,Q^2)}\right| \;=\; \frac{\sigma^{DD}_{\gamma*p}}{\sigma_t}\,. \tag{10}$$

Therefore directly from ZEUS data we can conclude:

$$\frac{|\Delta F_2|}{F_2} \;>\; 6\cdot 10^{-2}\,.$$

We would like to emphasize that after such an estimate we can start to discuss not just the question whether there is SC or not, but even whether we could describe the value of SC in our theory.

We would like also to note,that the details of the evolution equation for F_2^{DD} were discussed in ref.[8] and it was suggested to measure the sum

$$F_2 \,+\, F_2^{DD} \;=\; F_2(x_B,Q^2)\{\,1 \,+\, \frac{\sigma_t^{DD}}{\sigma_t}\,\}$$

in which all contributions of SC are cancelled.. Thus for this sum we can use the GLAP equation even in the region of very small x_B.

Using the explicit formula for $q+\bar{q}+G$ production for DD processes in ref. [8] we are able to estimate the value of r in the GLR equation directly from the ZEUS data. Indeed

$$\frac{\sigma^{DD}}{\sigma_t} \;>\; \frac{\alpha_s^2 81}{16}\cdot\frac{(xG(x,Q^2))|_{x=2x_B}}{R^2Q_0^2}\cdot\frac{1}{2}\frac{1}{2\omega_0}\,\frac{\ln^2\frac{Q^2}{Q_0^2}}{2}\;; \tag{11}$$

where the last term is the estimate for $q\bar{q}G$ production assuming that $xG(x,Q^2)\sim x^{-\omega_0},\omega_0\approx 0.3,\gamma_G\rightarrow\frac{1}{2}$. For simplicity I also integrated over the quark loop assuming Prytz's simplification [10] as well as one suggested by EKL [11]. Using $xG(x,Q^2=16)=15$ from HERA data and $Q_0^2=5GeV^2$ we can conclude from the above simple estimates that if the ZEUS collaboration really measured the value of the DD cross section in DIS $\mathbf{R^2>12GeV^{-2}}$.

The same conclusion we can get from the AKMS [12] estimates for SC (ΔF_2) in the GLR equation, namely the typical value for the ratio $\Delta F_2/F_2$ turns to be equal 10% for $R=R_{proton}$ while it is of the order of 30% for $R=\frac{1}{3}R_{proton}$.

To evaluate the importance of the result let me to recall you that there are two theoretical estimates for the value of R: from QCD sum rules [13] $R \approx \frac{1}{3} R_{proton}$ and dymamical correlations of gluons that I'll discuss later give $R \approx 0.32 Fm$ [14]. Let me also to recall that $R^2_{proton} = 25 GeV^{-2}$.

Thus we can state that if ZEUS really measured the value of the total cross section of DD process this data means the physical pucture with two radii : R_{proton} for constituent quarks and $R \ll R_{proton}$ for gluons is inconsistent. This is certainly the first signal for the death of the quark constituent model which for two decades has provided us sufficiently simple and accirate estimate for hadron - hadron collisions at high energy.

3.3 Saturation of the parton density or different physics for "soft" and "hard" processes?

The HERA data allow us to raise the question that in the title of this subsection. Indeed, we can separate two distinguished regions in Q^2 with quite different energy behaviour of the photoproduction cross section.

1. $Q^2 \ll 1\, GeV^2$

1. The total cross section is basically constant here (or increases slightly with energy).
2. $\frac{\sigma^{DD}}{\sigma_t} \approx 30\%$.

Thus in this kinematic region the photoproduction process with small photon virtuality looks like a typical "soft" process, as in hadron - hadron collision.

2. $Q^2 > 5\, GeV^2$

1. The total cross section increases rapidly with energy (x_B) $\sigma_t \propto x_B^{-\omega_0}$.
2. $\frac{\sigma^{DD}}{\sigma_t} \sim 10\%$.

Here we have of course the typical process of deeply inelastic scattering. Thus both experimental facts we can interpret as the contribution of the so-called "hard" Pomeron which is a solution to the BFKL equation [5] and gives $\sigma_t \propto x_B^{-\omega_0}$ with $\omega_0 > 0.4$ while the smallness of the ratio $\frac{\sigma^{DD}}{\sigma_t}$ has a natural explanation in small values of SC.

The question arises what is going on for intermediate value of photon virtualities $1 GeV^2 < Q^2 < 5 GeV^2$. We have two different scenarios for this kinematic region:

Landshoff picture: At $Q^2 < 1 GeV^2$ all experimental data for photopropduction as well as for other "soft" processes can be described by the exchange of a "soft" Pomeron which is the usual Regge pole with intercept $\alpha_P = 1 + \epsilon$ ($\epsilon \sim 0.08 \ll \omega_0$) (see ref.[15] for details). The small value of the ratio $\frac{\sigma^{DD}}{\sigma_t}$ can be interpreted as an indication that the SC are small and can be treated in a perturbation way in this approach. In particular Donnachie and Landshoff considered only two Pomeron exchange. In this picture the region of deeply inelastic scattering has different underlying physics related to a "hard" Pomeron and the transition between "hard" and "soft" regions look quite arbitrary at the moment. However this approach is very simple and provides an elegant description of all available experimental data on "soft" processes.

I would like to draw your attention to the fact that DD is a bigger fraction of the total inelastic cross section in DIS process than in typical hadron - hadron collisions (

note: this is one the most striking news of this conference !). This fact is very difficult to describe in the Donnachie - Landshoff approach.

Saturation of the parton density: The second scenario is intimately related to the hypothesis of the parton density saturation. In this scenario the "hard" Pomeron is responsible for the behaviour of the total gross section in both kinematic regions, but the small value of the ratio $\frac{\sigma^{DD}}{\sigma_t}$ we interpret differently for large and small virtualities: for large ones it supports the small value of SC while at small ones we interpret it as an indication that the SC becomes very large and leads to a black hadron (constituent quark) disc. The greatest advantage of this scenario is the unique description of the diffractive and inclusive processes based on solid theoretical background: properties of the "hard" Pomeron and the shadowing correction in perturbative QCD.

It is worthwhile to mention that strong SC gives rise simultaneously to the saturation of the parton density, the smooth behaviour of the total cross section at high energy, a natural explanation of the transition from steep energy behaviour of the "hard" Pomeron to smooth energy dependance of the "soft" total cross section and small value of the diffraction dissociation cross section in both kinematic regions (see ref. [16] for more information). However we have to note that the description of the data has not been done within this hypothesis in spite of the fact that these two scenarios give sufficiently different behaviour in the region of intermediate virtualities of photon. In the second scenario we expect some transition region with smooth behaviour of σ_t versus Q^2. The first try to extract this behaviour from available experimental data shows that such transition region does not contradict them [17] but it is too early to draw a deffinite conclusion from the data.

3 $\frac{1}{2}$ The EKL approach.

As a smooth transition to the purely theoretical section of my talk let me discuss here the EKL approach which is the first attempt to give a theoretical selfconsistent use of the HERA experimental fact that the data could be described only if we assume that $F_2 \propto x_B^{-\omega_0}$ at $Q^2 = 5GeV^2$.

The fact of such behaviour of the structure function in the region of small x_B has been predicted by the BFKL equation but the BFKL equation suffers from two major difficulties: 1) the next order corrections are big and 2) this equation is sensitive to the hypothesis that we made about the confinement. The first problem is technical in nature but the second is of principal importance, the worst thing that we even have not learned enough what kind of assumption about the confinement has been made in the BFKL equation.

The idea of the EKL paper is to understand better what we can get from the traditional approach for the low x_B behaviour of the structure functions and in what kinematic region we can trust this approach.

Let me start by recalling the main steps of our theoretical approach to deeply inelastic scattering:

1. We introduce the moments of the deep inelastic structure function,namely

$$M(\omega,r) \;=\; \int_0^1 x_B^{N-1} dx_B\, x_B G(x_B,Q^2) \;=\; \int_0^\infty e^{\omega y} dy [x_B G(x_B,Q^2)]\;, \tag{12}$$

where $\omega \;=\; N-1$, $y = \ln(1/x_B)$ and $r = \ln(Q^2/Q_0^2)$.
2. Each moment is given as Wilson Operator Product Expansion in the form:

$$M(\omega,r) \;=\; C_2(\omega,r)\langle p|O^{(2)}|p\rangle + \frac{1}{Q^2}C_4(\omega,r)\langle p|O^{(4)}|p\rangle + \ldots \frac{1}{Q^{i-2}}C_i(\omega,r)\langle p|O^{(i)}|p\rangle \ldots \tag{13}$$

where C_i is the coefficient function and $\langle p|O^{(i)}|p\rangle$ is the matrix element of the twist i operator . In such approach we absorbed all our unknowledge about confinement in the matrix element, in particular the power like behaviour of F_2 means that

$$\langle p|O^{(2)}|p\rangle \;=\; M(\omega, Q^2 = Q_0^2) \;\propto\; \frac{Z(\frac{Q_0^2}{\Lambda^2})}{\omega \;-\; \omega_0}\;. \tag{14}$$

3. It is well known from the renormalization group approach that a coefficient function C_i behaves as

$$C_i \;\propto\; e^{\gamma_i(\omega) r} \tag{15}$$

where γ_i is the anomalous dimension of the twist i operator [2].
4. Now we neglected all high twist contributions (all terms in eq. (1) except the first one) assuming that they are small at large value of Q^2 due to the factor $\frac{1}{Q^{i-2}}$ in front.
5. The anomalous dimension of the leading twist contribution can be calculated using GLAP evolution equation [1] and it is equal to

$$\gamma_2(\omega) \;=\; \frac{N_c \alpha_s}{\pi\omega} \quad at \quad \omega \to 0 \tag{16}$$

6. The BFKL equation can be rewritten as a prediction for anomalous dimension of leading twist operator in the form:

$$\gamma^G \;=\; \sum C_n \left(\frac{\alpha_s}{\omega}\right)^n \;=\; \frac{N_c\alpha_s}{\pi\omega} \;+\; \zeta(3)\left(\frac{N_c\alpha_s}{\pi\omega}\right)^4 \;+\; \ldots. \tag{17}$$

The main property of γ^G_{BFKL} is the fact that $\gamma^G_{BFKL} \;\to\; \frac{1}{2}$ when $\omega \;\to\; \omega_L \;\propto\; \alpha_s$ and the expression for ω_L is given in section 2.7.

7. In the EKL paper

$$\gamma \;= \gamma_0 \;+\; \gamma_1 \;+ \ldots +\; \gamma_n \;+\; \ldots$$

has been calculated assuming

$$\epsilon \;=\; \sqrt{\alpha_s(Q^2)} \;\leq\; \omega \;\ll\; 1 \tag{18}$$

taking into account the following orders of ϵ:

[2]For simplicity we consider here the case of fixed α_s.

γ	ϵ	ϵ^2	ϵ^3	ϵ^4
γ_0	$\frac{\alpha_s}{\omega}$	α_s	$\alpha_s\omega$	$\alpha_s\omega^2$
γ_1			$\frac{\alpha_s^2}{\omega}$	α^2
γ_3				$\frac{\alpha_s^3}{\omega^2}$
γ_4				$\frac{\alpha_s^4}{\omega^4}$

It should be stressed that the whole approach can be justified only if $\omega_0 > \omega_L$.

The x_B behaviour of the structure function can be obtained by performing the inverse Mellin transform;

$$F_2(x_B, Q^2) = \frac{1}{2\pi i}\int_C d\omega e^{(\gamma(\omega)t+\omega y)} M(\omega, Q^2 = Q_0^2)\ , \tag{19}$$

where $y = -\ln x_B$ and $t = \ln\frac{Q^2}{Q_0^2}$ and the contour is to the right of all singularities in M as well as to the right of the saddle point (ω_S) which can be found from the equation:

$$\frac{d}{d\omega}\{\omega y + \gamma(\omega)\, t\}|_{\omega=\omega_S} = 0\ . \tag{20}$$

We have two very different cases: 1) $\omega_S > \omega_0$ and 2) $\omega_S < \omega_0$. In the first case we can evaluate the integral using the steepest decent method which gives a result equivalent the so-called Double Log Approximation of pQCD. In the second case we can close the contour on the pole $\omega = \omega_0$ and we get the simple result that

$$F_2(x_B, Q^2) = F_2(x_B, Q^2 = Q_0^2)e^{\gamma(\omega_0)t}\ . \tag{21}$$

Fig. 3 shows that if $\omega_0 \approx 0.5$ in HERA kinematic region we have the simple answer of eq. (21) for $x_B < 10^{-2}$.

The $\gamma(\omega)$ calculated by EKL in ref. [11] is given in Fig. 4. From this picture one can see how well the ELK approach could work. It is very important to realize that only if the value of ω_0 that incorporates the unknown confinement dynamic in the EKL approach is large enough, namely $\omega_0 > \omega_L \approx 0.35$ we can trust this approach. If this is not the case we have to be reconciled with the fact that the confinement forces are important even in typical "hard" process such as DIS at low x_B. Fortunately we have the BFKL equation that descibes the main qualitative features of low x_B interaction in DIS but we certainly need more transparent and more formal understanding of what hypothesis about confinement has been incorporated in this equation.

The second possible solution of the problem is the different scenario for what could happen at low x_B. Indeed, if the screening corrections described by the GLR equation (8) enter into the game before the difference between the BFKL and the GLAP equation becomes visible, the situation turns out to be very attractive from a theoretical point of view since in the GLR equation the unknown confinement forces has been included only in one nonperturbative (phenomenological ?) parameter R, which has very transparent physical meaning of the correlation length of gluons inside a hadron at $x_B \sim 1$.

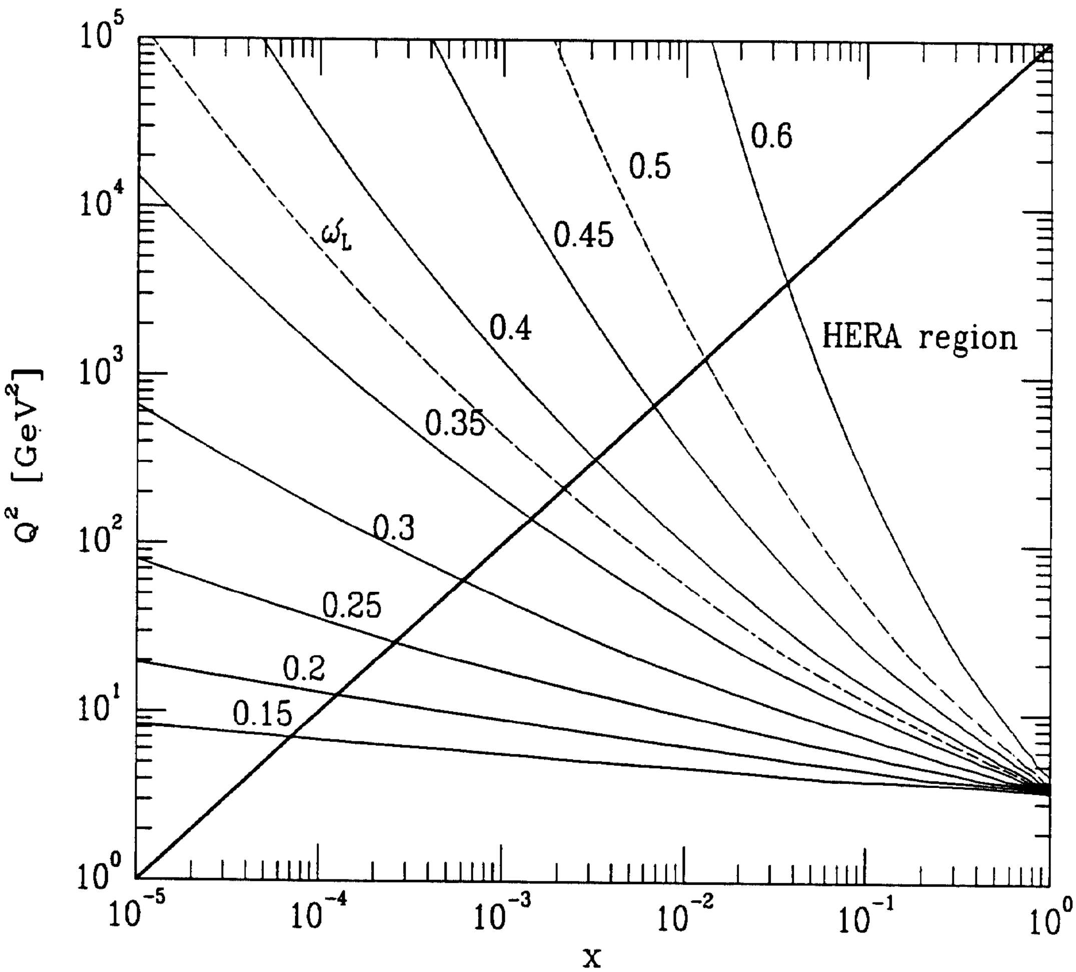

Figure 3: Contour plot showing fixed values of ω_S in Q^2, x_B plane [11].

4 On the way to an analytic solution.

In this theoretical section of my talk I am going to discuss only three selected topics which I hope will be able to clear up our typical difficulties and achievements in this field, namely : i) a new understanding of the BFKL equation, ii) dynamic gluon correlations and iii) "hard" diffraction dissociation processes.

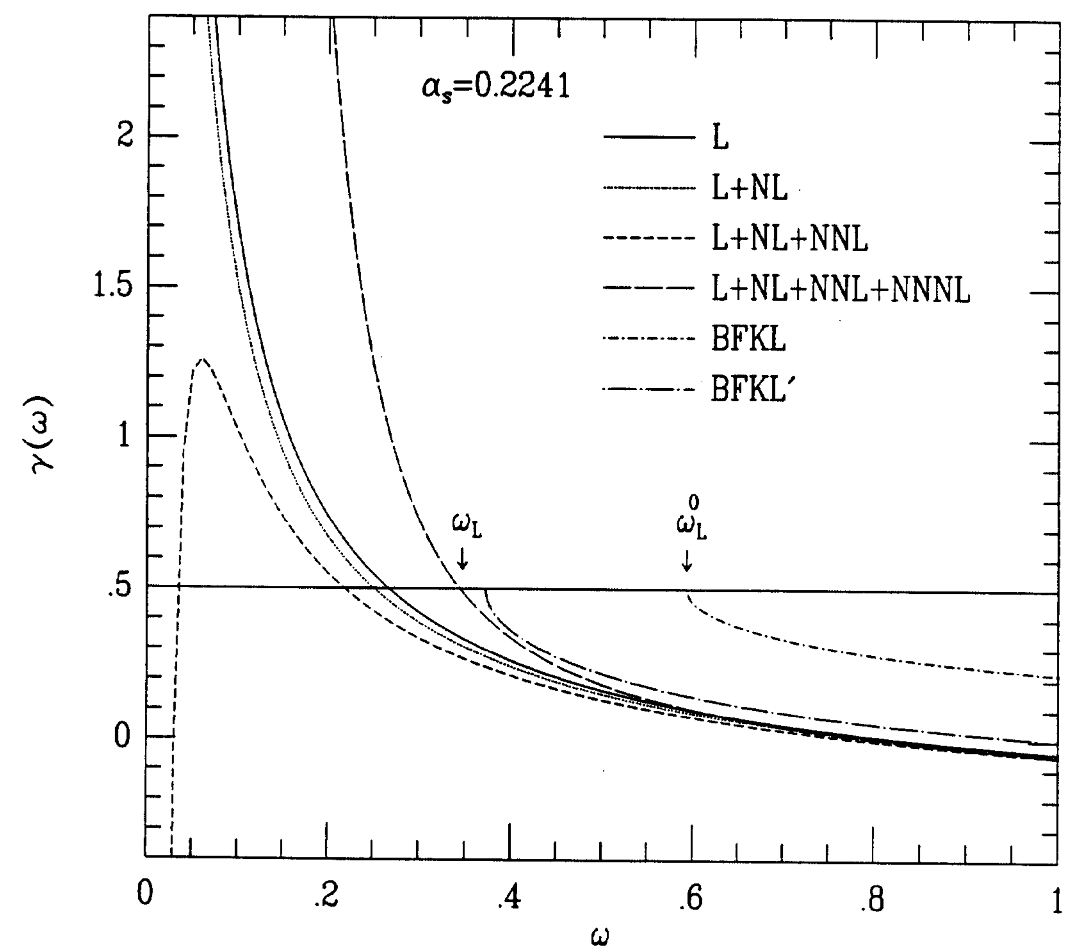

Figure 4: The GG anomalous dimension in various approximations.

4.1 New understanding of the BFKL equation.

I hope that I have convinced you how important it bis to understand better the physical meaning and the formal grounds of the BFKL equation. I firmly believe that during this year A. Mueller [18] (and N.N. Nikolaev with collaborators but six months later [19]) has achieved the considerable progress in both understanding and formal derivation of the BFKL equation and its generalization. Mueller's main idea is to construct the small x infinite momentum partonic wavefunction of a hadron in QCD while BFKL calculated the amplitude for n - gluon production in so called multireggeon kinematic region. The wavefunction gives us much richer information on the hadron interaction, has very transparent physical meaning and makes the bridge between our parton approach to "hard" processes and new phenomena that we anticipate in the region of high density QCD.

1.

The technical trick that has been used is also very instructive, namely it turns out that the wavefunction looks much simpler in the mixed representation in which each parton is labeled by its fraction of the total hadron momentum x_i and the transverse coordinate r_{ti}. The transverse coordinate is especially useful since in the low x_i region the i^{th} gluon can be considered as being emitted from the system of $i-1$ partons with spatial transverse coordinates of these "sources" being frozen during the emission of $i-th$ gluon. Thus we can consider these of $i-1$ partons as a system of $(i-1)$ $q\bar{q}$ dipoles since each gluon can be viewed as quark - antiquark pair if number of colours N_c is big enough. So the only thing that one needs to write down is the emission of the $i-th$ gluon by such a system of dipoles. This problem has been solved in ref.[18]. For example for emission of gluon $(x_2, r_{tG} = r_2)$ from one dipole which is the quark $(x_q = 1 - x_1, r_{tq} = r_0 = 0)$ and antiquark $(x_{\bar{q}} = x_1, r_{t\bar{q}} = r_1)$ is equal to

$$\psi^{(1)}(x_1, x_2; r_1, r_2) \;=\; -\frac{igT^a}{\pi}\psi^{(0)}(x_1; r_1)\{\frac{r_{2\lambda}}{r_2^2} - \frac{r_{21\lambda}}{r_{21}^2}\} \cdot \epsilon_2^\lambda \,. \tag{22}$$

2.

Mueller made one very important step in our understanding of our parton system, namely he found what sum rule plays the role of the momentum sum rules in the GLAP approach for low x_i partons. This sum rules is the normalization of the partonic wave-function:

$$\int \prod^n \frac{dx_i}{x_i} \prod^n d^2 r_{ti} |\Psi(x_1, ... x_n; r_{t1}, ... r_{tn})|^2 \;=\; 1 \,. \tag{23}$$

Using this equation one can easily take into account so called virtual corrections, which in this case are mostly known as gluon reggeization or non -Sudakov form factor. The importance of this step can be compared only with the transition from Gribov - Lipatov form of usual evolution equation with Sudakov form factor in the kernel to well known Lipatov - Altarelli - Parisi elegant form based on direct use of the momentum sum rules in QCD.

3.

The physical application of this new approach has not been considered but Mueller noted at his Durham talk [20] that his approach will be able to resolve the old problem with the BFLK equation. Indeed, the physical meaning of the growth of the structure function at $x_B \to 0$ is the increase of the number of "wee" partons (N) that can interact with the target ($N \propto x_B^{-\omega_0}$)(see, for example, review [21]). However the multiplicity of gluons calculated as the ratio $\int \frac{Ed^3\sigma}{dA^3 p_{jet}}/\sigma_t$ turns out to be small (of the order of $\alpha_s \ln \frac{1}{x_B}$). It means that partons are in a very coherent state in a typical inelastic event. However the behaviour of the parton cascade at large multiplicity should be quite different from Poisson distribution since at large multiplicity all N parton can be freed in the interaction.

4.2 Dynamic gluon correlations.

In refs. [22] [23] was shown that we oversimplified the problem when we assumed that probability of annihilation is proportional to ρ^2 (see eq. (6)) in the derivation of the GLR equation (see section 2.6). Strictly speaking this probability is proportional to P_2, the probability to find two parton in one cell of the parton cascade. It turns out that the ratio

$$\frac{P_2}{\rho^2} \propto e^{\frac{1}{(N_c^2-1)^2}\sqrt{\frac{16 N_c \alpha_s}{\pi}\ln\frac{Q^2}{Q_0^2}\ln\frac{1}{x_B}}} \tag{24}$$

increases with x_B. It means that we have to take into account such sort of dynamic correlations which can crucially change the GLR equation. In a more formal approach the dynamic correlations originate from high twist contributions in the Wilson Operator Product Expansion (see eq. (13)) which can be rewritten as follows:

$$x_B G(x_B,Q^2) \;=\; x_B G^{(1)}(x_B,Q^2) + \frac{1}{Q^2} x_B^2 G^{(2)}(x_B.Q^2) \ldots + \ldots \frac{1}{Q^{2(n-1)}} x_B^n G^{(n)}(x_B,Q^2) \ldots \tag{25}$$

where $P^{(n)} = \frac{x_B^n G^{(n)}}{(\pi R^2)^n}$ and $P^{(1)} = \rho$. A The value of the anomalous dimension of the high twist operators has been found in ref. [24] by E.Laenen, E.Levin and A.Shuvaev. The main idea was to reduce the complicated problem of the gluon - gluon interaction to interaction of colourless gluon - " ladders" (Pomerons) in the t-channel. It was shown in refs. [22] [23] that this idea works for the case of the anomalous dimension of the twist four operator. The fact that we can consider the rescattering of n - pomerons to find the anomalous dimension γ_{2n} really means that we are dealing with a quantum mechanical problem: the calculation of ground state energy for an n - particle system where the interactions are attractive and given by a four particle contact term (λ). We can calculate the value of λ in QCD. It turns out that

$$\lambda \;=\; 4\,\bar{\alpha}_s\,\delta\,. \tag{26}$$

This observation considerably simplifies the problem and will enable us to reduce it to solving the Nonlinear Schrodinger Equation for n-Pomerons in t - channel. It is very important to mention that the effective theory is a two dimendsional one or in other words the Schrodinger equation can be written for n - particles moving only in one dimension. It is well known (see refs. [25] for details) that this problem can be solved exactly.

The answer for the energy of the ground state translated into the value of the anomalous dimension of the twist $2n$ operator is the following:

$$\gamma_{2n} \;=\; \frac{\bar{\alpha}_s n^2}{\omega}\{1 \;+\; \frac{\delta^2}{3}(n^2 \;-\; 1)\}\;, \tag{27}$$

where $\delta = (N_c^2 - 1)^{-1}$. We can trust the answer only when $\frac{\delta^2}{3} \ll 1$. So we first need to find the generalization of the GLR equation (eq. (6)) to clear up what value of n

really is important for the deep inelastic structure function using the above result for the anomalous dimension and only after the solution we have to go back to the calculation of the value of the anomalous dimension.

The point is that our all perturbation series are asymptotic ones, so I know only one practical way how we can operate with such series, namely to we find (if possible) the analytical function with the same series and treat this function as a solution to our problem. Expanding this function we are able to study what value of typical n works in the series and to consider the question whether we can trust our answer. If the value of the typical n will be of the order of 1 we can claim that we have solved our problem, if not we have to go back and try to find a more general expression for the value of the anomalous dimension of high twist operators that is valid for any large n.

E.Laenen and E.Levin (the paper is still in preparation) have obtained the generalization of the GLR evolution equation, taking into account both the arbitrary initial condition and the exact value of the anomalous dimension (see eq. (27)).

The first step in such generalization was to write down the equation for $P^{(n)}$ using the idea of competition of two processes in our parton cascade: emission and annihilation that has been used for derivation of the GLR equation and has been discussed in section 2.6. It is easy to understand that the equation for $P^{(n)}$ looks as follows:

$$\frac{\partial^2 \, P^{(n)}}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} \; = \; \omega \gamma_n \, P^{(n)} \; - \; n \cdot \frac{\alpha_s^2 \gamma}{Q^2} \, P^{(n+1)} \; , \tag{28}$$

Since the contribution of the high twist operators become essential in the region of small x_B we have to consider the whole series (25) and using the above infinite set equations we can try to get the equation for the deep inelasatic structure function.

To get the equation we introduce the generating function

$$g(x_B, Q^2, \eta) \; = \; \sum_{n=1}^{\infty} e^{n\eta} g^{(n)} \; , \tag{29}$$

where $g^{(n)} \; = \; x_B^n G^{(n)}(x_B, Q^2)$. Comparing g with eq. (25) we see that the deep inelastic function is equal to

$$x_B G(x_b, Q^2) \; = \; Q^2 \, g(x_B, Q^2, \eta = -\ln Q^2) \; . \tag{30}$$

Using the generalized GLR equation (28) we can easily get the equation for generating function g that sums the infinite set of equations (28):

$$\frac{\partial^2 g(x_B . Q^2, \eta)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} \; = \; \bar{\alpha}_s g''_{\eta\eta} \, + \, \frac{\bar{\alpha}_s \delta^2}{3} (g''''_{\eta\eta\eta\eta} \, - \, g''_{\eta\eta}) \, - \, \gamma \, e^{-\ln Q^2} \, e^{-\eta} \, (g'_\eta \, - \, g) \; , \tag{31}$$

where $g'_\eta \; = \; \frac{\partial g}{\partial \eta}$ and γ is the same as in the GLR equation (6).

To solve the above equation we need to impose some initial and boundary conditions which are the price we must pay for the relative simplicity of the equation. The boundary condition looks very simple :

$$\eta = fixed;\ \ln\frac{1}{x_B} = fixed;\ \ln Q^2 \to \infty\ g(x_B, Q^2, \eta) \to e^{\eta}\, g_{LLA}(x_B, Q^2)\ , \tag{32}$$

where g_{LLA} is the solution of usual GLAP evolution equation.

However the initial condition is a much more complicated problem since we need to know the fuction $g(x_B = x_{B0}, Q^2, \eta)$, while experimentally we are only able to measure the structure function. So we need more detail information about the structure of a hadron in the region $x_B \sim 1$. To start, we suggest the initial condition in the form:

$$g(x_{B0}, Q^2, \eta) \ = \ \sum_{n=1}^{\infty} e^{n\eta}\frac{(-1)^n}{n!}\cdot [g_{LLA}(x_{B0}, Q^2)]^n \ = \ 1 \ - \ exp(-e^{\eta} g_{LLA}(x_{B0}, Q^2))\ . \tag{33}$$

In favour of the above formula we can say that it is simple, has a very transparent physical meaning, namely,it reflects the assumption that there is no correlation between gluons with $x_B \sim 1$ except the fact that they are distributed in the hadron disc of the radius R. In the case of the nucleus such an approach can be proved and corresponds to the so-called Glauber Theory of shadowing correction. In the case of the deeply inelastic scattering the formula of this type was discussed by A.Mueller in ref. [26] and we use formulas from his paper to establish the exact relationship with g_{LLA} in eq. (15).

We are only in the begining of finding of the solution to eq. (31). At the moment we can claim that we found how eq. (31) transforms to nonlinear GLR equation if we neglect the second term in r.h.s. of eq. (31) and assume the eikonal initial condition of eq. (33). We also solve eq. (31) with eq. (33) in the oversimplified case only remaining the second term in r. h. s. of eq.(31). The result looks very encouraging since the effective n that works in the series of eq.(25) turns out to be of order 1. However we certainly have to consider this result as very preliminary since we need to understand the general solution of eq. (31) better.

4.3 "Hard" DD.

Here I am going to discuss one particular process, namely "hard" DD in which a jet with large transverse momentum $p_t \gg Q \gg m$ is produced in the diffraction dissociation. I chose this process not because it is extremely interesting but rather because this process can illustrate all problems in QCD in the most direct way.

First,let me remind you how the calculation of this process looked in the good old Reggeon Approach. The key idea of the theoretical approach was Mueller's technique [27] in which the DD can be described by two Reggeon diagrams (see Figs. 5.1 and 5.2): the first gives the contribution which is similar to inclusive production in the typical inelastic process while the second is a new contribution in comparison with inclusive production: the emission from a vertex. However about twenty years ago in refs. [28] it was shown that unfortunately we have to take into account the third contribution of Fig. 5.3 type. This new diagram meant that there were no AGK cutting rules for DD, we had no so-called

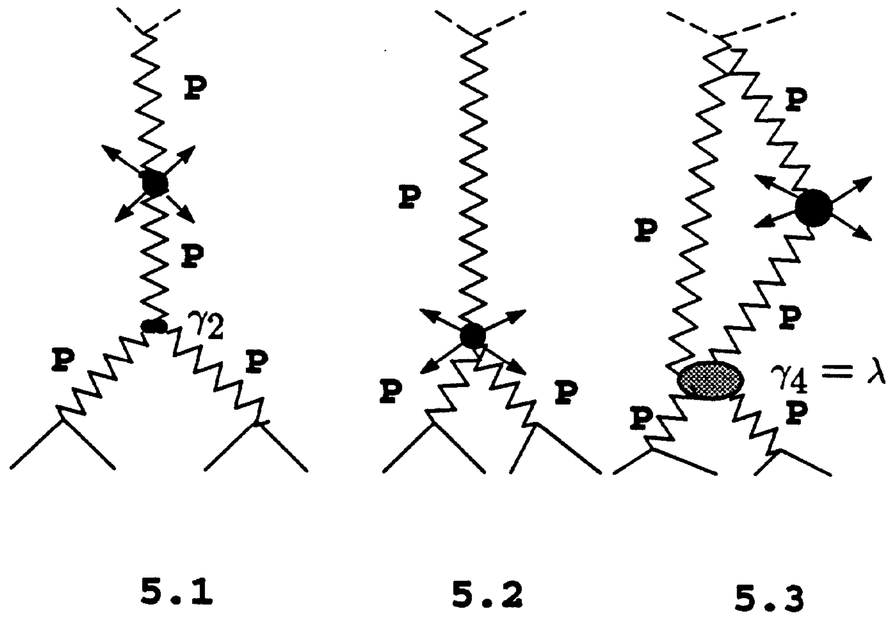

Figure 5: The DD in "good old" Reggeon Approach.

Reggeon Field Theory and no way to construct a selfconsistent theoretical approach based on the reggeons and their interactions. Thus the result of refs. [28] led to the demise of the Reggeon Approach as a way to built a theory for high energy interaction.

Now I can formulate the question what is happening with the DD dissociation in QCD and for simplicity let us consider the "hard" diffraction in DIS because we can apply perturbative QCD to this process and use the factorization theorem [30], that plays the role of AGK cutting rules in QCD. It is easy to guess what we want,namely we hoped that in QCD only the first two diagrams of Fig.5 contribute. The answer for QCD has been obtained by Bartels, Levin and Wuesthoff (BLW) recently (the paper is still in preparation) and it is shown in Fig.6. Unfortunately it turns out that we have all three contributions: the first one corresponds to the Ingelman - Schlein mechanism [29] and gives us a tool to study so called Pomeron structure function, the second one is the emission from a vertex and as far as I know was firstly suggested by Ryskin [31] and the third is the most dangerous and unpleasant contribution that was suggested by Frankfurt and Strikman [31] and called coherent diffraction (CD) by them. Thus QCD does not help and we have all the old problems in QCD too. However, we have two pieces of good news in QCD: the contribution is small for DIS ($\propto \frac{1}{Q^2}$) and the high energy asymptotic of CD is proportional to $\frac{1}{N_c^2-1} \ll 1$. BLW also understood how the factorization theorem works for inclusive hard processes and why it fails in DD (see also the paper of Collins,Frankfurt

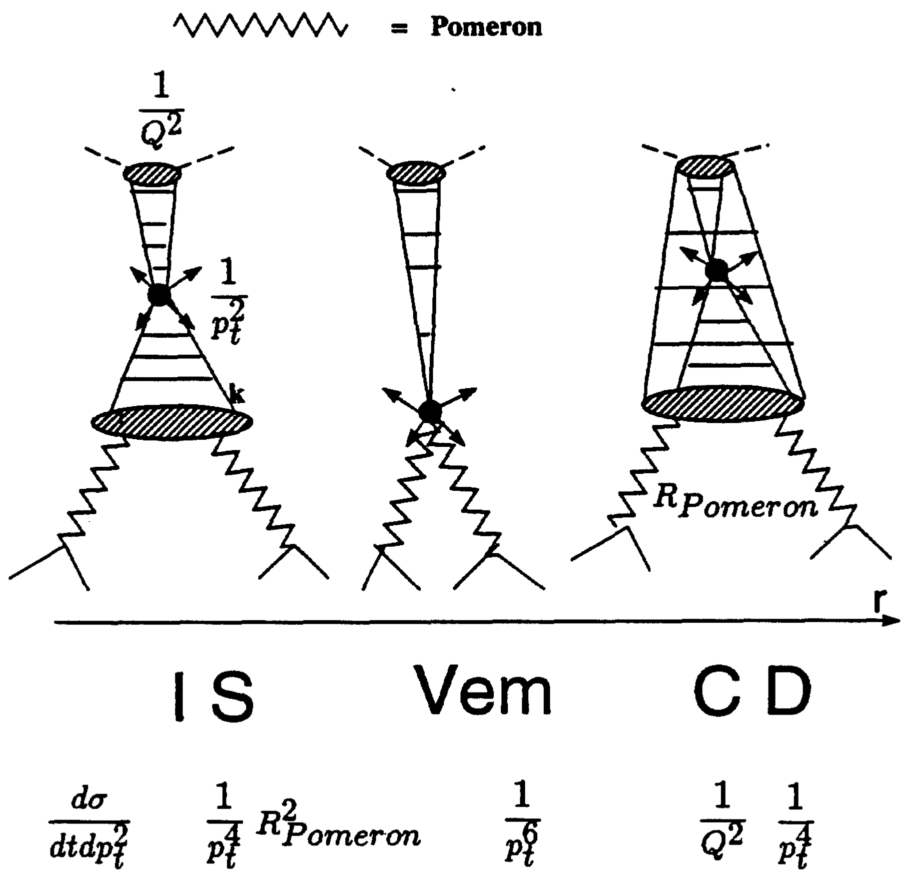

Figure 6: The DD in DIS in QCD.

and Strikman [31] where this problem has been discussed on general ground).

5 Tragedy of low x_B.

Now let me share with you the elements of a tragedy that I see in the present status of low x_B physics:

• QCD has not cured any difficulty that we had in Reggeon Approach. The only profit that we have had from QCD is the fact that the interaction between four "ladders" (Pomerons) is small ($\propto \frac{1}{N_c - 1} \ll 1$).

• The field becomes too complicated even for guys whom have had 20 years daily experience in the manipulation of Feyman diagrams.

• The demise of the SSC essentially diminishes our hope for direct measurement of the low x_B (high density QCD) phenomena. The only information can come from HERA, Fermilab and perhaps LHC (?). The first data from HERA are rather in favour of the sufficiently small SC or in other word they could be interpreted as the indication that we started only to touch the interesting kinematical region at HERA.

I do not want that you interpret my above remarks as too pessimistic. The conclusion that I want to draw is that we need to search for some new approach to theoretical solution of low x_B problem less based on Feynman diagrams that is suited for perturbative calculation and more related to nonperturbative methods and physical intuition. Fortunately I can already say something on possible way out of the perturbative approach.

Effective Lagrangian for high energy QCD. The first attempt to develop nonperturbative approach to low x_B problems was to write down the effective Lagrangian for the region of small x_B. Intrinsically we assume that such a Lagrangian should be simpler that the QCD one and allows one to apply some direct numerical procedure (lattice calculation for example) to calculate the scattering amplitude with this Lagrangian. It should be stressed the attempts to calculate the amplitude with full QCD Lagrangian have failed by now. At the moment we have two effective theories on the market: one was proposed by Lipatov [32] which looks not much simpler that full QCD but it certainly incorporates all results of perturbative calculations, and the second was suggested by Verlinde and Verlinde [33], which is much simpler and is suited for lattice -like calculation but it has not been checked how well this effective theory describes the perturbative results. Moreover there is some indication [34] that this theory cannot describe the virtual correction in the BFKL Pomeron.

Thermodynamics of high density QCD. I firmly believe that we need to write down the correct kinetic equation for high density QCD. Such an approach has certainly at least one big advantage: the smooth matching with the GLR equation. Unfortunately we have not yet understood how to write such an equation in our nonequilibrium situation. However we have understood better the physical meaning of the new typical momentum in our parton cascade ($\langle|p_t|\rangle$ in section 2.7). It turns out that this momentum is the Landau - Pomeranchuk momentum for our parton medium [21]. Thus the gluon emission with transverse momentum less that $\langle|p_t|\rangle$ is small due to destructive interference between emission before and after collision of the parton with other partons in the medium. The experience with the calculation of the anomalous dimension of high twist gluonic operators also give us understanding why for bosonic degrees of freedom such as a gluon there could be saturatioon of gluon density. Indeed, our cascade is rather a one dimensional one and for such a system the direction of motion plays the role of spin for fermions.

Let me finish my talk with the very optimistic statement that certainly we have learned more about low x_B parton system both theoretically and experimentally during this year after the Durham workshop. Unfortunately we have learned so many things that we had no time to swim and have fun in Eilat.

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